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STRUCTURAL DIVISION

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AN ANALYTICAL PROCEDURE FOR CALCULATING TRUSS DISPLACEMENTS

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SYNOPSIS

An analytical procedure for the calculation of displacements and changes in slope in a plane structure subjected to loads and displacements in its plane is presented. The procedure is based on the concept that the computations of geometry involving small angle changes and displacements are interchangeable with those of statics. The method is simple and may be used to determine the displacement of a number of points in a trussed structure in any direction by means of one set of calculations.

INTRODUCTION

Various methods, both analytical and graphical, are available for the calculation of displacements for a trussed structure. Among these are those methods based either on the geometry of deformations or on the principle of conservation of energy. The method of virtual work or Castigliano's second theorem may be used to determine the displacement of a single point of a structure in any direction. The Williot-Mohr diagram is a convenient procedure for finding the displacements of all the joints in a truss; however, the procedure is graphical and its accuracy is limited by the scale used. The bar-chain method (sometimes called elastic-weight method or angle change method) may be used to find the displacements of a number of points in a truss; but, is limited to bar chains which do not have vertical members.

In this paper a generalized analogy is presented for calculating the displacements and changes in slope of plane structures loaded in their own plane. The analogy allows the calculations of movements to be made by substituting equations of statics for those of geometry. The generalized analogy is illustrated by applications to trussed structures.

Geometry of Small Angle Changes and Displacements and Its Relation to the Equations of Statics

The relationship that exists between the laws of statics and that of geometry of small angles and displacements is shown in figures 1 and 2. Consider that a line with its original position aob, figure 1, is subjected to an angle change, $\Delta\phi_o$, at 0 and a small length of this line, s, changes in length by an amount equal to Δs_o. If ao remains fixed in position, point b will move to b'.

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If the angle change and displacements are small, the following results may be written:

Rotation of Tangent at b = $\Delta\theta_b = \Delta\phi_o$ ----------(1)
Displacement in x direction at b = $\Delta x_b = y\Delta\phi_o + \Delta s_o \cos\theta_o$ ---(2)
Displacement in y direction at b = $\Delta y_b = x\Delta\phi_o + \Delta s_o \sin\theta_o$ ---(3)

Assume that the line aob, figure 1, is a plan view of a cantilever fixed at b, loaded by a force, $F_o = \Delta\phi_o$ perpendicular to and out of the plane of the paper, and a moment, $M_o = \Delta s_o$ about an axis parallel to the tangent at o. The force and moment are shown vectorially in figure 2; then, the shear at b and the moments about the x and y axes at b are equal to the change in slope and the displacements in the x and y directions at point b of the line shown in figure 1.

From the foregoing analogy the computations of statics and geometry are interchangeable, and the following statements may be made:[3]

(a) An angle change or rotation about an axis may be treated as a force along the axis of rotation.
(b) A displacement along any axis may be treated as a moment about the axis of displacement.

Generalized Analogy for a Plane Structure Subjected to Loads and Displacements in its Own Plane

The analogy shown in figure 1 and 2 is generalized and applied to any plane structure subjected to loads and displacements in its own plane in figure 3 and 4.

A portion of a plane structure which has an original position ab is shown in figure 3 and due to some external effect, such as loads in the plane of the structure, temperature change, etc., this member deforms and displaces to position a'b'. During this movement each end of the member has experienced a change in slope and a displacement, and along the axis of the member axial unit strains and angle changes about axes normal to the plane of the member have taken place. These terms with their proper sign convention are defined in figure 3. The slope of the member θ at any point is taken as positive when it is up and to the right.

Equations (4), (5), and (6) give the change in slope and displacement in the x and y direction at the right end of the member in terms of the change in slope and displacement at the left end plus angle changes and axial unit strains along the member. The angle change and axial unit strain may vary as a function of s; the x and y distances are taken as positive in all cases.

$$\Delta\theta_b = \Delta\theta_a + \int_a^b d\phi \qquad (4)$$

$$\Delta x_b = \Delta x_a + \Delta\theta_a y_a + \int_a^b y\,d\phi + \int_a^b \delta\,ds\cos\theta \qquad (5)$$

$$\Delta y_b = \Delta y_a + \Delta\theta_a x_a + \int_a^b x\,d\phi + \int_a^b \delta\,ds\sin\theta \qquad (6)$$

3. "Laterally Loaded Plane Structures and Structures Curved in Space," by Frank Baron and James D. Michalos, ASCE Transactions 1952, Paper No. 2493, pp. 279-314.

A conjugate or analogous member which in plan view follows the centroidal axis of the true member ab shown in figure 3 is depicted in figure 4. This member is loaded perpendicular to its own plane by a distributed load, f, and by applied moments per unit length, m, which act about the longitudinal axis of the member. The member is held in equilibrium by shears and moments about x and y axes at each end of the member. Moments are shown vectorially and the right hand rule is used to obtain their proper sense.

By applying the three equations of statics for this type of loading, equations (7), (8), and (9) may be written as follows:

$$V_b = V_a + \int_a^b f\,ds \tag{7}$$

$$M_{xb} = M_{xa} + V_a y_a + \int_a^b f\,ds\,y + \int_a^b m\,ds\cos\theta \tag{8}$$

$$M_{yb} = M_{ya} + V_a x_a + \int_a^b f\,ds\,x + \int_a^b m\,ds\sin\theta \tag{9}$$

It is apparent that these equations are identical to equations (4), (5), and (6), respectively, if the substitution of the analogous terms tabulated in figures 3 and 4 are used.

This generalized analogy enables one to calculate displacements and changes in slopes in a plane structure loaded in its own plane by means of a conjugate member which is loaded normal to its own plane.

Assuming that the angle changes and unit strains along the member can be found for the loading condition, in the true member, there remain six unknowns which are the displacement components and changes in slope at each end of the true member. Since there are only three equations of statics that may be used for the conjugate member to solve for these unknown displacement components and changes in slope, it can be seen that at least three of the six unknowns must be found from the boundary conditions at each end of the member. For a member between supports, at least three of the six displacement components are known from the boundary conditions at the supports.

Calculation of Displacements and Changes in Slope in Trussed Structures

The procedure for applying this analogy to the solution of a trussed structure will be outlined for the truss shown in figure 5a; it is desired to determine the vertical and horizontal displacements of all joints.

A stress analysis of the structure enables one to calculate the stress in each member and thence the axial unit strain in each member. For the conjugate member, or bar chain as it is often called, shown in Fig. 5b a continuous line between the supports and containing the joints for which the displacements are desired, is selected. There is a change in slope of member ad but no displacement in the true structure at point a; therefore at end a of the conjugate member only a shear can exist. There is a change in slope of member ce and a horizontal displacement of point c in the true structure; therefore at end e of the conjugate member a shear and a moment about the x-axis must exist. The conjugate member has applied loads F_d, F_b, F_e normal to the plane of the paper which are equal to the angle changes at the same points in the true structure. The value of these angle changes can be calculated easily as will be shown subsequently. In addition, the conjugate member is subjected to applied moments per unit length m_{ad}, m_{db}, m_{be}, and m_{eb}, which are

equal to the axial unit strains in the true structure. These moments are shown vectorially in Fig. 5b. The conjugate member must be in equilibrium under the applied known loads and moments and the end reactions. Since there are three equations of statics available, V_a, V_c, and M_{xc} may be found by statics. Knowing these quantities the displacement of any point in the true structure in a desired direction may be found by finding the moment at this point in the conjugate member about an axis in the direction of the desired displacement. Thus the horizontal and vertical displacement of each joint may be found. If desired the change in slope of any member in the true structure at a particular point may be found by finding the shear at this point in the conjugate member.

Other conjugate members or bar chains could be selected such as abdec or adebc and though the angle changes would be different depending on the case considered, the final results for the displacement of the joints would be the same.

Calculation of Concentrated Angle Changes at Joints of Trussed Structures

One of the requirements of the solution just described was that the angle changes at d, b, e in the true structure which became the forces F_d, F_b, F_e in the conjugate member be known.

These angle changes can be found by writing geometric expressions for the triangles of members which make up the true structure. However, the generalized analogy may be used to find these angle changes. Suppose the angle change at d used in the preceding solution was to be calculated in this manner. The closed triangle adb, is taken as the true member. The axial unit strains in members ad, db, ba are known but the angle change at a, d, b are unknown. The conjugate member, shown in figure 5c then becomes a closed triangle, adb, subjected to applied moments per unit length m_{ad}, m_{db}, m_{ba} which are equal to known axial unit strains in the true member. These vectors are directed in a clockwise direction if the axial strain is tension or plus, and counter clockwise for compression. In addition the conjugate member is subjected to forces F_a, F_d, F_b normal to the plane of the paper which equal the unknown angle changes at a, d, and b, respectively, in the true member.

Since the conjugate member is a closed figure it must be in equilibrium under the moments and forces indicated. There are three unknowns, F_a, F_d, and F_b, and three equations of statics available for their solution.

A force F out of the plane of the paper indicates that the angle change is an increase or opening of the angle and a force into the plane of the paper indicates a decrease or closing of the angle. Using this procedure the angle changes to be used in the solution of truss displacements may be determined.

If, in solving for the displacements of the truss in figure 5a, a conjugate member adec were used, the total angle change at d would be obtained by summing the angle change at d in triangle adb and the angle change at d in triangle bde.

Numerical Examples

Applications of the generalized analogy to the calculations of displacements and angle changes for a trussed structure are given in the following examples. Example 1 is a numerical solution for the truss of figure 5a loaded as indicated in figure 6; example 2 is the solution for displacements in a three-hinged

arch, figure 7; and, example 3 illustrates the application to a tower type structure, illustrated in figure 8.

Example 1: The loads and the resulting axial forces in each member are shown on a two panel Warren truss in figure 6a. For simplicity assume that the product of AE is unity for all members; then the unit strain in each member equals the value indicated in figure 6a. The conjugate member selected is shown in figure 6b. Initially for the conjugate member only the values of m_{ad}, m_{db}, m_{be}, and m_{eb} are known. However, the angle changes at joints d, b, and e are computed in figure 6c, and the reactions at a and c are computed in figure 6d. Thus the value of all the forces and moments on the conjugate member may be found. The horizontal and vertical displacements are then determined in figure 6e and 6f. In taking moments about any axis, the contribution of an applied moment per unit length such as m_{ad}, is simply the value per unit length multiplied by the projected length of the moment vector on the axis about which moments are being taken. Note that in the calculation of the angle changes in figure 6c, F_b is found to be a force into the plane of the paper. This indicates a closing or decrease of the angle. When this force is shown on the conjugate member of figure 6b it is indicated as coming out of the plane of the paper to conform with the sign convention of counterclockwise rotation when going from left to right which was established in figure 3 and figure 4.

Example 2: A three-hinged arch subjected to a vertical and a horizontal load is shown in figure 7a. The axial force in each member together with the cross-sectional area of each member are indicated in this figure. It is desired to find the vertical and horizontal displacements of joints b, c, and d. Stresses are shown in figure 7b. In the true structure there is no displacement at point a or at point e, but a change in slope of member ab or ed is possible. Therefore for the conjugate member shown in figure 7c only shears exists at points a and e. The angle change at c remains as an unknown reaction in the conjugate member. Thus the conjugate member has three unknown reactions V_a, V_e, R_c. These are calculated by using the three equations of statics as shown in figure 7e. With all the forces and moments acting on the conjugate member known the horizontal and vertical displacements of points b, c, and d may be found by calculating the moments in the conjugate member at each of these points which act about horizontal and vertical axes. This is done in figure 7f and figure 7g.

Example 3: A tower type structure subjected to a single vertical load is shown in figure 8a and in this figure the axial force in each member together with the cross-sectional area of each member is indicated. Stresses are shown in figure 8b. It is desired to find the vertical and horizontal displacements of the joints indicated in figure 8c. At joint a of the true structure there is no vertical or horizontal displacement possible. Also since each end of member am rests on a support there can be no change in slope of this member. Thus in the conjugate member there will be no moment and no shear at end a. At joint h of the true structure there will be vertical and horizontal displacement, and there can also be a change in slope of member gh. Thus in the conjugate member there can exist at end h, moments about x and y axes and a shear. The forces acting on the conjugate member on the remaining points between a and h are angle changes in the true structure which may be found as described in the previous examples. Only the results are indicated for this example. The conjugate member thus has three unknown reaction components which may be found by the equations of statics.

The solution for this problem is carried out in a tabular form in figure 8d. The shear indicated in Col. 4 gives the change in slope of each member while the vertical and horizontal displacement for each joint are indicated in Cols. 13 and 14.

Calculations for Displacements and Changes in Slope in Beams. The Method of Conjugate Beam.

An application of the generalized analogy to problems of beams under load immediately leads to the method of conjugate beam as demonstrated in figure 9. The beam under load experiences angle changes along its length which may be defined as $d\phi/ds = M/EI$, if shearing deformations are neglected. These angle changes are shown as a distributed load acting normal to the plane of the paper on the conjugate member of figure 9c. The boundary condition at the left end of the true member indicates that a change in slope but no displacement may take place, thus only a shear, V_a, exists in the conjugate member at the left end. At the right end of the true member there is a change in slope and a horizontal displacement possible, thus in the conjugate member a shear, V_b, and a moment, M_{xb}, may exist at the right end. By inspection M_{xb} must be zero. V_a and V_b in the conjugate member may be found by statics and thence the shear and moment at any section of the conjugate member will give the change in slope and displacement in the true member. Figure 9d indicates the way in which the conjugate member is commonly shown when it is a beam.

Calculations of Displacements and Changes in Slope in Rigid Frames and Arches

A further application of the method is shown in figure 10 a for a statically determinate rigid frame subjected to a lateral load, P. The frame deforms as indicated and at any section there may exist an axial force, a shear, and bending moment. If the effect of shear and axial force on displacements and changes of slope in the structure are neglected then displacements and changes in slope will be produced by angle changes due to M/EI.

Figure 10b depicts the conjugate structure, F_1, F_2, F_3, and F_4 being known loads normal to the plane of the paper which are the resultants of the M/EI diagrams (therefore the angle changes) shown. Since a change in slope may take place at a and d in the true structure, shears must exist at a and d in the conjugate structure. And since a displacement parallel to the plane of the rollers at d may take place in the true structure, a moment about the axis of displacement must be shown at d in the conjugate structure.

In the conjugate structure, which must be in equilibrium under the forces and moment shown in figure 10b, there are three unknown reactions and three equations of statics available for these solutions. Once the reactions are known, the shear at any section (which represents change in slope in the true structure) and moment at any section about a desired axis (which represents displacement in the direction of that axis in the true structure) may be found.

If desired the effect of axial force and shear in the true structure on displacements and changes in slope may be included. The axial force produces axial unit strain in the true structure which would be indicated as an applied moment per unit length about the longitudinal axis of the conjugate structure. The shear produces an angle change in the true structure which could be added to that produced by bending moment and indicated as a force normal to

the plane of the paper in the conjugate structure. These effects may be of importance in arch calculations.

The concept of the conjugate structure for a rigid frame has been pointed out by a number of investigator;[4,5,6,7] its inclusion in this paper, along with the conjugate beam method, is merely to emphasize that both are specific examples of the application of the generalized analogy described earlier in this paper.

CONCLUSION

The presented analogy uses the principle that laws of geometry for small angles are interchangeable with those of statics. This principle is applied to the determination of displacements and of angle changes in a trussed structure. The method is simple and permits the computation of more than one displacement by one set of calculations.

The various examples cited are a few applications of the method. Other extensions may include the solution of secondary stress problems and determination of displacements in a space frame.

The writers believe that this generalized analogy is a powerful analytical tool. Through its use engineers may visualize structural action while using the principles of statics for the determination of geometric changes in a structure.

4. "The Column Analogy," by Hardy Cross, Bulletin No. 215, University of Illinois Engineering Experiment Station.
5. "The Conjugate Frame as a Tool for Evaluating Deflections," by Glenn Murphy, Proc. VII Int. Congress of Applied Mechanics, London, 1948, p. 131.
6. "On Plane Rigid Frames Loaded Perpendicular to Their Own Plane," by W. Prager and G. E. Hay, Quarterly of Applied Math., 1943, p. 49.
7. "Analysis of Multi-Span Rigid Frames," by J. S. Kinney in "Steel Rigid Frames," by Martin Korn, J. W. Edwards, Inc., Publishers.

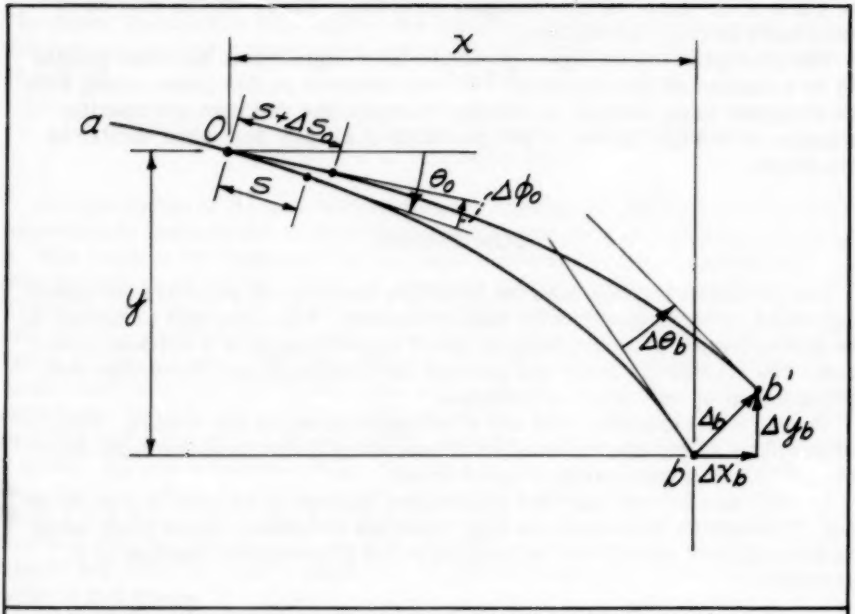


Fig. 1-- Geometry of Small Angles and Displacements.

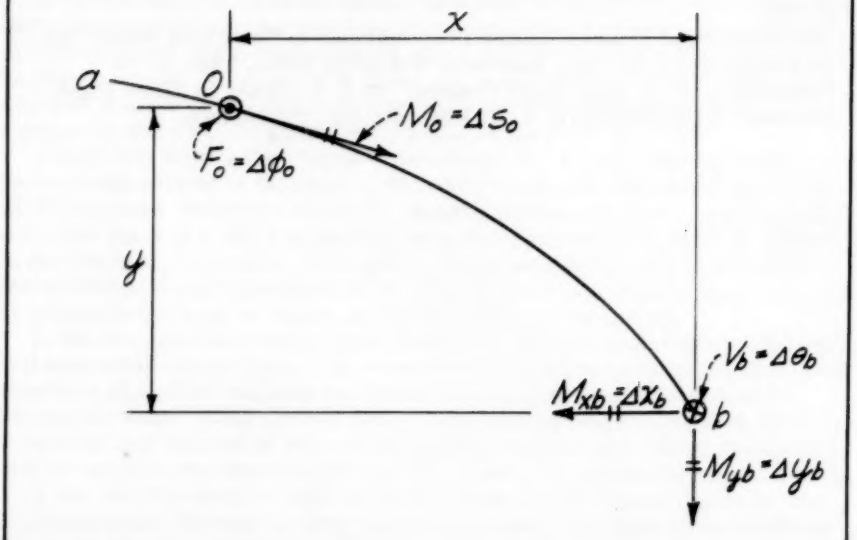
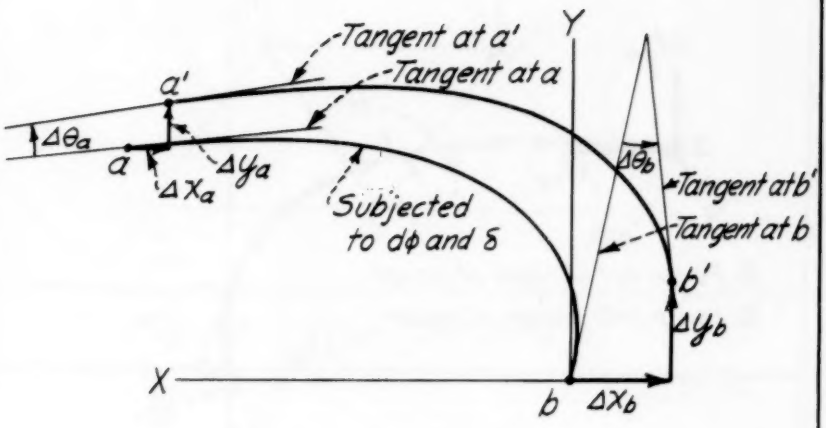


Fig. 2-- Free Body Diagram of a Member Loaded Normal to its Own Plane.



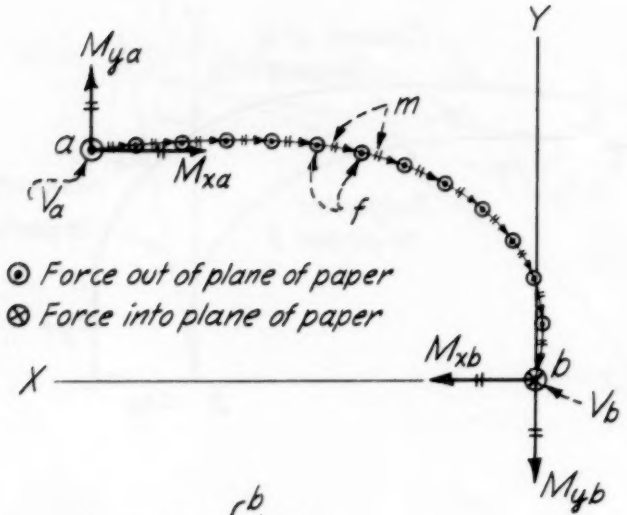
$$\Delta\theta_b = \Delta\theta_a + \int_a^b d\phi \qquad (4)$$

$$\Delta x_b = \Delta x_a + \Delta\theta_a y_a + \int_a^b y\,d\phi + \int_a^b \delta\,ds\cos\theta \qquad (5)$$

$$\Delta y_b = \Delta y_a + \Delta\theta_a x_a + \int_a^b x\,d\phi + \int_a^b \delta\,ds\sin\theta \qquad (6)$$

Term	Symbol	Sign Convention
Angle change	ϕ	Counterclockwise (+)
Angle change per unit length	ϕ/s	Counterclockwise (+)
Axial unit strain	δ	Tension (+) Compression (-)
Change in slope	$\Delta\theta$	Counterclockwise rotation of tangent is (+)
Displacements	Δx Δy	Up or to the right is (+)

Fig. 3--True Member.



$$V_b = V_a + \int_a^b f\,ds \qquad (7)$$

$$M_{xb} = M_{xa} + V_a y_a + \int_a^b f\,ds\,y + \int_a^b m\,ds\cos\theta \qquad (8)$$

$$M_{yb} = M_{ya} + V_a x_a + \int_a^b f\,ds\,x + \int_a^b m\,ds\sin\theta \qquad (9)$$

Term	Symbol	Sign Convention
Force ⊥ to plane of member	F	Force out of plane of paper is (+) ⊙
Force per unit length ⊥ to plane of member	f	Force out of plane of paper is (+) ⊙
Applied moment per unit length about longitudinal axis of member	m	Moment vector pointing away from starting point at left end is (+); towards starting point at left end is (−)
Shear at section	V	At right end V into plane of paper is (+) At left end V out of plane of paper is (+)
Moment at section about x or y axis respectively	M_x M_y	At right end moment vectors down or to the left are (+). At left end moment vectors up or to the right are (+)

Fig. 4 -- Conjugate Member.

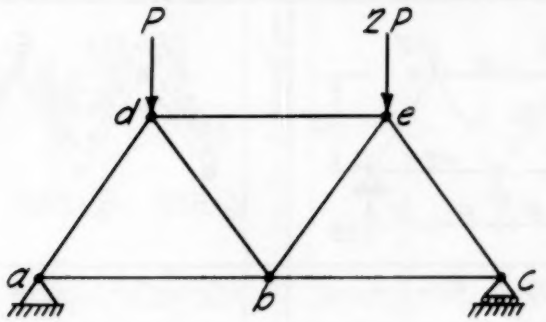


Fig. 5a -- True Structure.

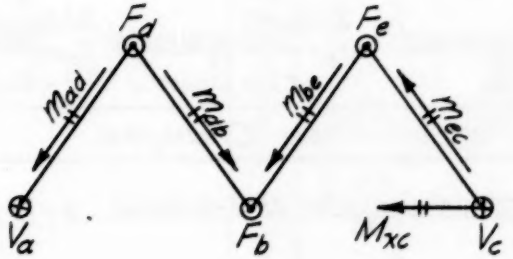


Fig. 5b -- Conjugate Member.

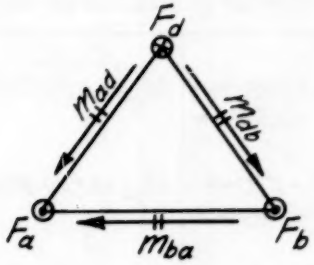


Fig. 5c -- Conjugate Member for Finding Angle Changes at a, d, and b.

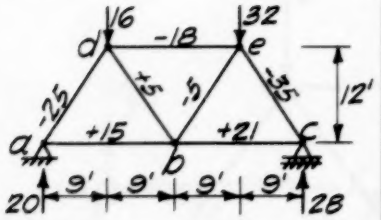


Fig.6a--True Structure.

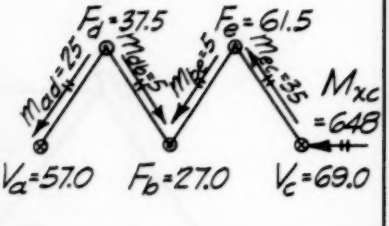
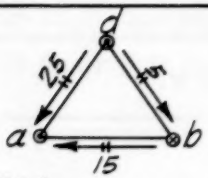


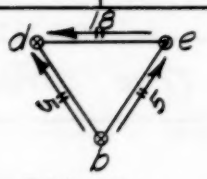
Fig.6b--Conjugate Member.



$\Sigma M_{ab}=0$

$F_d = \frac{25(9)+15(18)-5(9)}{12}$

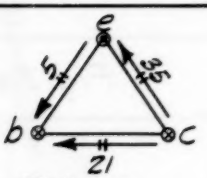
$= 37.5$ Up



$\Sigma M_{de}=0$

$F_b = \frac{5(9)-5(9)+18(18)}{12}$

$= 27.0$ Down



$\Sigma M_{bc}=0$

$F_e = \frac{35(9)+21(18)+5(9)}{12}$

$= 61.5$ Up

Fig.6c--Angle Changes.

$\Sigma M_{yc}=0$

$V_a = \frac{37.5(27)+27(18)+61.5(9)-(25+5+5-35)(12)}{36} = 57.0$ Down

$\Sigma M_{ya}=0$

$V_c = \frac{37.5(9)+27(18)+61.5(27)-(25+5+5-35)(12)}{36} = 69.0$ Down

$\Sigma M_{xc}=0$

$M_{xc} = (37.5+61.5)12-(25-5+5+35)9 = 648 \leftarrow\!\!\!

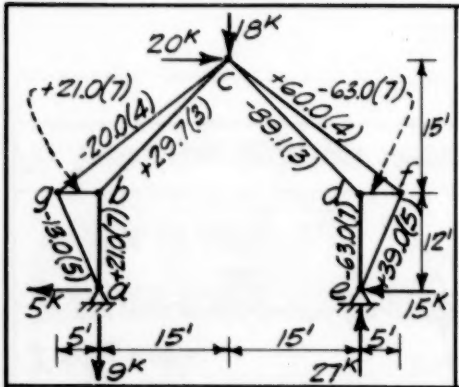


Fig. 7a -- True Structure (Forces and Member Areas)

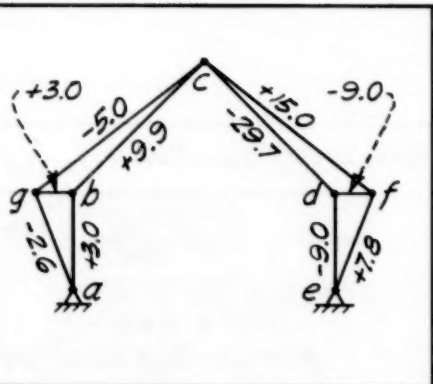


Fig. 7b -- Stresses in Members (KSI)

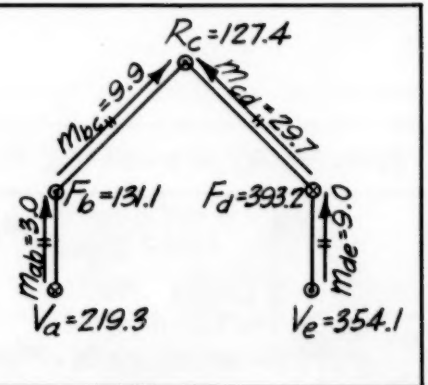


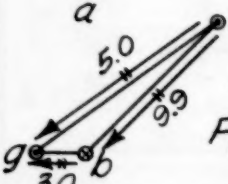
Fig. 7c -- Conjugate Member.

$\Sigma M_{ag} = 0$

$F_b' = \frac{3.0(13)+2.6(13)}{4.62} = 15.8$ KSI down

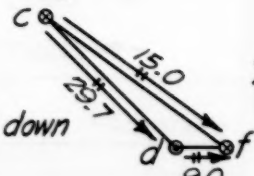
$\Sigma M_{ef} = 0$

$F_d' = \frac{9.0(13)+7.8(13)}{4.62} = 47.3$ KSI up



$\Sigma M_{cg} = 0$

$F_b'' = \frac{3.0(4)+9.9(21)+5.0(25)}{3} = 115.3$ KSI down



$\Sigma M_{cf} = 0$

$F_d'' = \frac{9.0(4)+29.7(21)+15.0(25)}{3} = 345.9$ KSI up

Fig. 7d -- Angle Changes.

$\Sigma M_{ae} = 0$

$R_c = \frac{(393.2-131.1)(12)+(29.7-9.9)(15)}{27}$

$= 127.4$ KSI (up)

$\Sigma M_{ed} = 0$

$V_a = \frac{131.1(30)+127.4(15)+(3+9)(12)+(9.9+29.7)(15)}{30}$

$= 219.3$ KSI (down)

$\Sigma M_{ab} = 0$

$V_e = \frac{393.2(30)-127.4(15)+(3+9)(12)+(9.9+29.7)(15)}{30}$

$= 354.1$ KSI (up)

Fig. 7e--Reactions for Conjugate Member.

$M_{xb} = 219.3(12)$

$= 2632$ KSI (FT) ⟵

$\Delta_{xb} = \frac{2632}{E}$ FT →

$M_{xc} = 219.3(27)-131.1(15)+9.9(15)$

$= 4103$ KSI (FT) ⟵

$\Delta_{xc} = \frac{4103}{E}$ FT →

$M_{xd} = 219.3(12)+127.4(15)$

$-(29.7-9.9)(15)$

$= 4246$ KSI (FT) ⟵

$\Delta_{xd} = \frac{4246}{E}$ FT →

Fig. 7f--Horizontal Deflections.

$M_{yb} = 3.0(15)$

$= 45$ KSI (FT) ⇓

$\Delta_{yb} = \frac{45}{E}$ FT ↑

$M_{yc} = (219.3-131.1)15-3.0(12)-9.9(15)$

$= 1138$ KSI (FT) ⇑

$\Delta_{yc} = \frac{1138}{E}$ FT ↓

$M_{yd} = (219.3-131.1)30-127.4(15)$

$-3.0(12)-(29.7+9.9)(15)$

$= 108$ KSI (FT) ⇑

$\Delta_{yd} = \frac{108}{E}$ FT ↓

Fig. 7g--Vertical Deflections.

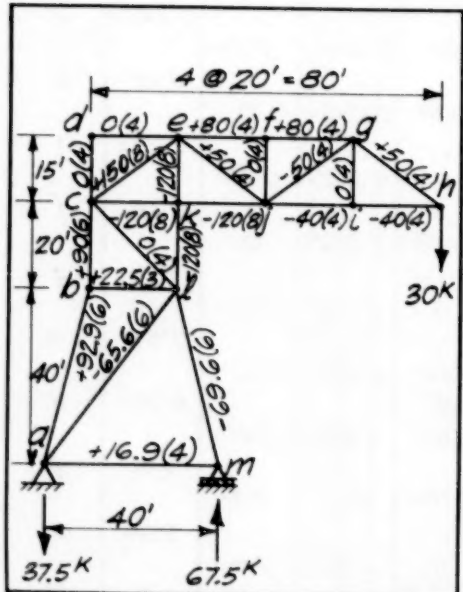


Fig. 8a--True Structure. (Forces and Member Areas)

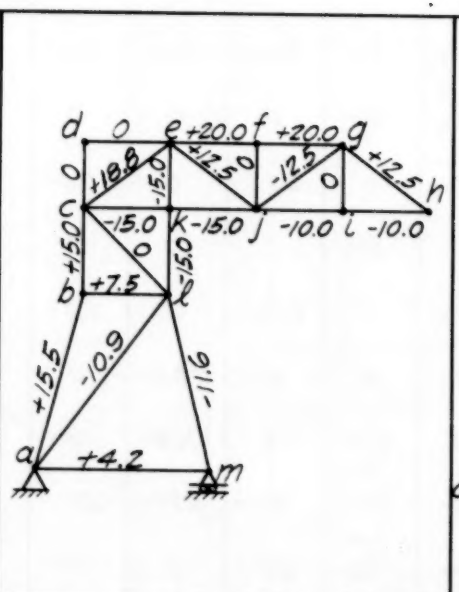


Fig. 8b--Stresses in Members. (KSI)

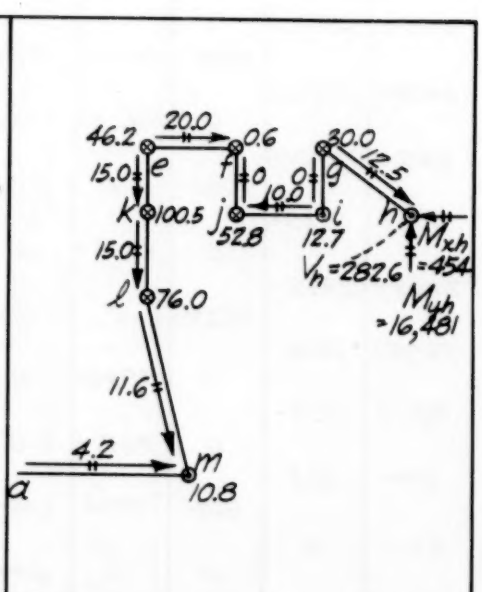


Fig. 8c--Conjugate Member.

1	2	3	4	5	6	7 = 3×5	8 = 3×6	9	10	11 = 4×9	12 = 4×10	13	14
POINT	$F=E(\Delta\phi)$	$m=E(\delta)$	$V=E(\Delta\theta)$	$L\sin\theta$	$L\cos\theta$	$mL\sin\theta$	$mL\cos\theta$	y	x	Vy	Vx	$M_y=E(\Delta y)$	$M_x=E(\Delta x)$
	KSI	KSI	KSI	FT	FT	KSI(FT)	KSI(FT)	FT	FT	KSI(FT)	KSI(FT)	KSI(FT)	KSI(FT)
a	0											0	0
		+4.2	0	0	+40	0	+168	0	+40	0	0		
m	+10.8											0	+168
		-11.6	+10.8	+40	-10	-464	+116	-40	-10	-432	-108		
l	-76.0											-572	-148
		-15.0	-65.2	+20	0	-300	0	-20	0	+1304	0		
k	-100.5											-872	+1156
		-15.0	-165.7	+15	0	-225	0	-15	0	+2486	0		
e	-46.2											-1097	+3642
		+20.0	-211.9	0	+20	0	+400	0	+20	0	-4238		
f	-0.6											-5335	+4042
		0	-212.5	-15	0	0	0	+15	0	-3188	0		
j	-52.8											-5335	+854
		-10.0	-265.3	0	+20	0	-200	0	+20	0	-5306		
i	+12.7											-10641	+654
		0	-252.6	+15	0	0	0	-15	0	+3789	0		
g	-30.0											-10641	+4443
		+12.5	-282.6	-15	+20	-188	+250	+15	+20	-4239	-5652		
h	+282.6											-16481	+454

Fig. 8d -- Tabular Solution for Vertical and Horizontal Displacements.

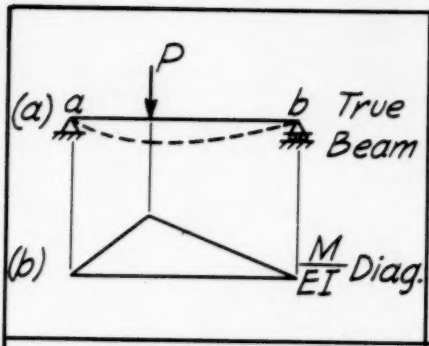


Fig.9a,b--True Beam.

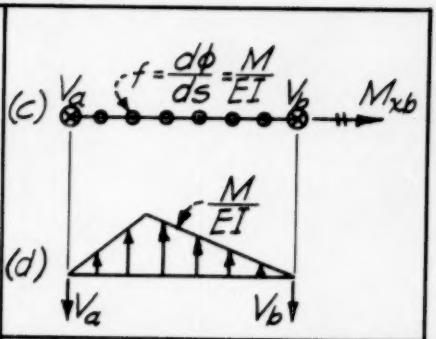


Fig.9c,d--Conjugate Beam.

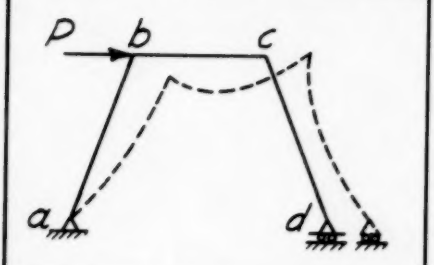


Fig.10a--True Structure.

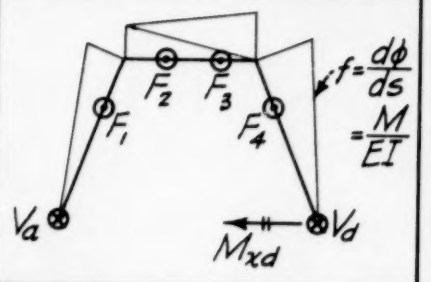


Fig.10b--Conjugate Structure.

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